Name solutions

September 21, 2011

ECE 311 Exam 1 Fall 2011

Closed Text and Notes

1) Be sure you have 12 pages.

2) Write only on the question sheets. Show all your work. If you need more room for a particular problem, use the reverse side of the same page.

3) no calculators allowed

4) Write neatly, if your writing is illegible then print.

5) The last 2 pages contain equations that may be of use to you.

6) You can leave π and ε_o in your answers.

7) This exam is worth 100 points.

(6 pts) 1.a) Convert the point ($\sqrt{2}$, $\sqrt{2}$, $\sqrt{2}$) in Cartesian to cylindrical coordinates.

$$\rho = \sqrt{x^2 + y^2} = \sqrt{2+2} = 2$$

$$\phi = \tan^{-1}\frac{y}{x} = \tan^{-1}\frac{\sqrt{2}}{\sqrt{2}} = \tan^{-1} 1 = 45^\circ \text{ or } \frac{\pi}{4}$$

$$z = \sqrt{2}$$

$$\left(2, \frac{\pi}{4}, \sqrt{2}\right)$$

b) Convert the point $(1, \frac{\pi}{4}, \frac{\pi}{4})$ in spherical to Cartesian coordinates.

$$\rho = r \sin\theta = 1 \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$x = \rho\cos\phi = \frac{\sqrt{2}}{2}\cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}\frac{1}{\sqrt{2}} = \frac{1}{2}$$

$$y = \rho\sin\phi = \frac{\sqrt{2}}{2}\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}\frac{1}{\sqrt{2}} = \frac{1}{2}$$

$$z = r\sin\theta = 1\sin\frac{\pi}{4} = 1\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\left(\frac{1}{2}, \frac{1}{2}, \frac{\sqrt{2}}{2}\right)$$

(4 pts) 2. In cylindrical coordinates, what unit normals are tangent to $\rho = 1$m.

$\hat{a}_\phi$ and $\hat{a}_z$

(5 pts) 3. What geometry is described by the intersection of the surfaces $\theta = \frac{\pi}{3}$ and r = 1m.

circle

(5 pts) 4. A 30 μC charge experiences a force of $\mathbf{F} = 3\hat{\mathbf{a}}_\mathbf{x} + 6\hat{\mathbf{a}}_\mathbf{y} - 9\hat{\mathbf{a}}_\mathbf{z}\ N$. What is the electric field?

$$\vec{E} = \frac{\vec{F}}{Q} = \left(\frac{3}{30}\hat{a}_x + \frac{6}{30}\hat{a}_y - \frac{9}{30}\hat{a}_z\right) \times 10^6 \ \frac{N}{C}$$

$$= \left(1\hat{a}_x + 2\hat{a}_y - 3\hat{a}_z\right) \times 10^5 \ \frac{N}{C}$$

$$= \left(1\hat{a}_x + 2\hat{a}_y - 3\hat{a}_z\right) \times 10^5 \ \frac{V}{m}$$

$$\frac{N}{C} = \frac{N}{C}\frac{m}{m} = \frac{J}{Cm} = \frac{V}{m}$$

(6 pts) 5. a) what are the units of the electric flux density field?

$$\frac{C}{m^2}$$

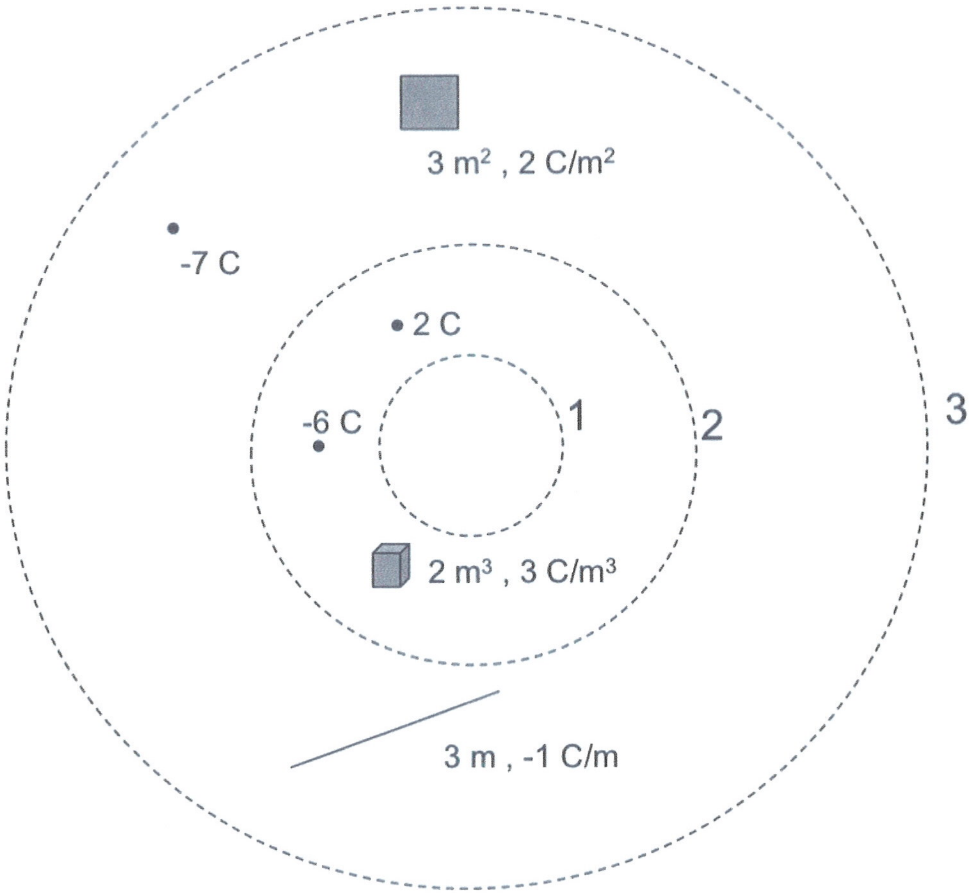
b) what are the units of the electric field intensity?

$$\frac{V}{m}$$

c) What are the units of the scalar potential field?

$$V$$

(9 pts) 6. In the following figure the dashed lines represent closed spherical surfaces that completely surround any objects shown within. Determine the following integrals.



$\oint \mathbf{D} \bullet \mathbf{dS}$ over surface 1 = 0

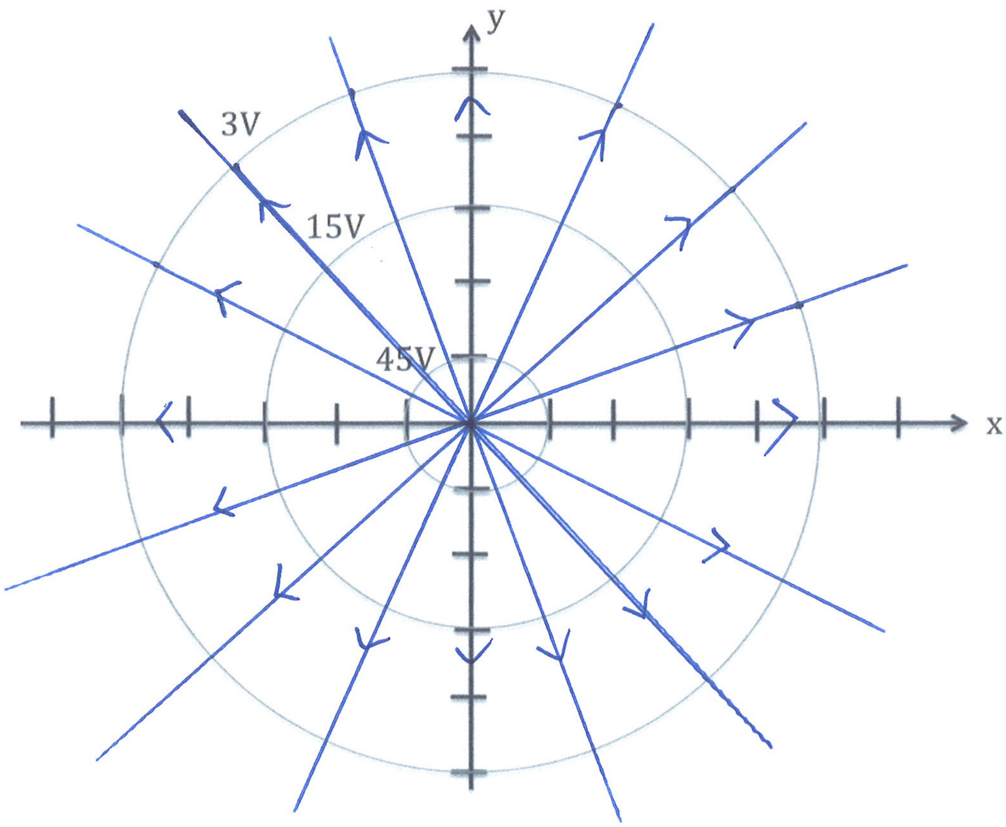
$\oint \mathbf{D} \bullet \mathbf{dS}$ over surface 2 = $2C - 6C + (2m^3)(3\, C/m^3) = 2C$

$\oint \mathbf{D} \bullet \mathbf{dS}$ over surface 3 = $\oint_2 \vec{D} \cdot \vec{ds} + (3m)(-1\frac{C}{m}) + (3m^2)(2\frac{C}{m^2}) - 7C$

$= 2C - 3C + 6C - 7C$

$= -2C$

(15 pts) 7. Shown are some equipotential surfaces caused by a point charge at the origin. Each tick on the x- and y-axis represent 1 m.



(10 Pts) a) Determine the value of the point charge at the origin.

for a point charge at the origin $\quad V(r) - V(\infty) = \dfrac{Q}{4\pi\epsilon_0 r}$

for $r = 1m \quad 45V - V(\infty) = \dfrac{Q}{4\pi\epsilon_0 (1m)}$

for $r = 2m \quad 15V - V(\infty) = \dfrac{Q}{4\pi\epsilon_0 (3m)}$

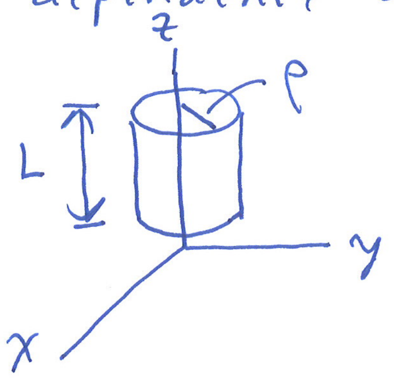
subtracting $\quad 30V = \dfrac{Q}{6\pi\epsilon_0\, m}$

$$Q = (30V)(6\pi)\frac{10^{-9}}{36\pi}\frac{F}{m}\,m = 5\times10^{-9}\,C = 5nC$$

(5 pts) b) on the figure indicate the electric field intensity. (Just a qualitative sketch.)

(10 pts) 8. Along the z-axis is an infinite line of charge of density $\rho_L = \frac{10^{-9}}{36\pi} C/m$. Use Gauss' law to determine the electric field.

The field will have cylindrical symmetry with only an $\hat{a}_\rho$ component and no dependence on ϕ or z.



use a cylindrical Gaussian surface centered on the z-axis with length L and radius ρ.

$$\oint \vec{D}\cdot d\vec{s} = \underbrace{\int_{top} \vec{D}\cdot d\vec{s}}_{=0} + \underbrace{\int_{bottom} \vec{D}\cdot d\vec{s}}_{=0} + \int_{side} \vec{D}\cdot d\vec{s}$$

$$\int_{side} \vec{D}\cdot d\vec{s} = 2\pi\rho L D_\rho = Q_{enclosed} = \left(\frac{10^{-9}}{36\pi}\frac{C}{m}\right) L$$

$$D_\rho = \frac{1}{2\pi\rho L}\left(\frac{10^{-9}}{36\pi}\frac{C}{m}\right) L = \frac{1}{2\pi\rho}\left(\frac{10^{-9}}{36\pi}\frac{C}{m}\right)$$

$$E_\rho = \frac{D_\rho}{\epsilon_0} = \frac{1}{\left(\frac{10^{-9}}{36\pi}\frac{F}{m}\right)}\frac{1}{2\pi\rho}\left(\frac{10^{-9}}{36\pi}\frac{C}{m}\right) = \frac{1}{2\pi\rho}\frac{C}{F}$$

$$= \frac{1}{2\pi\rho} V = \frac{1}{2\pi\rho}\frac{V}{m}$$ since ρ is a distance in m

$$\vec{E} = \frac{1}{2\pi\rho}\hat{a}_\rho \frac{V}{m}$$

(10 pts) 9. An electric field in free space is given as $\mathbf{E} = x\hat{\mathbf{a}}_x + 4z\hat{\mathbf{a}}_y + 4y\hat{\mathbf{a}}_z \ \frac{V}{m}$. If V(1,1,1) = 5 V, determine V(1, 2, 2).

For a static field $\int \vec{E} \cdot \vec{dl}$ is independent of path. So the path I will take is $(1,1,1) \to (1,2,1) \to (1,2,2)$

$$V(1,2,2) - V(1,1,1) = -\int_{(1,1,1)}^{(1,2,1)} \vec{E} \cdot dy\,\hat{a}_y - \int_{(1,2,1)}^{(1,2,2)} E \cdot dz\,\hat{a}_z$$

$$V(1,2,2) - 5V = \left[-\int_{1}^{2} 4z\,dy \Big|_{z=1} - \int_{1}^{2} 4y\,dz \Big|_{y=2} \right] V$$

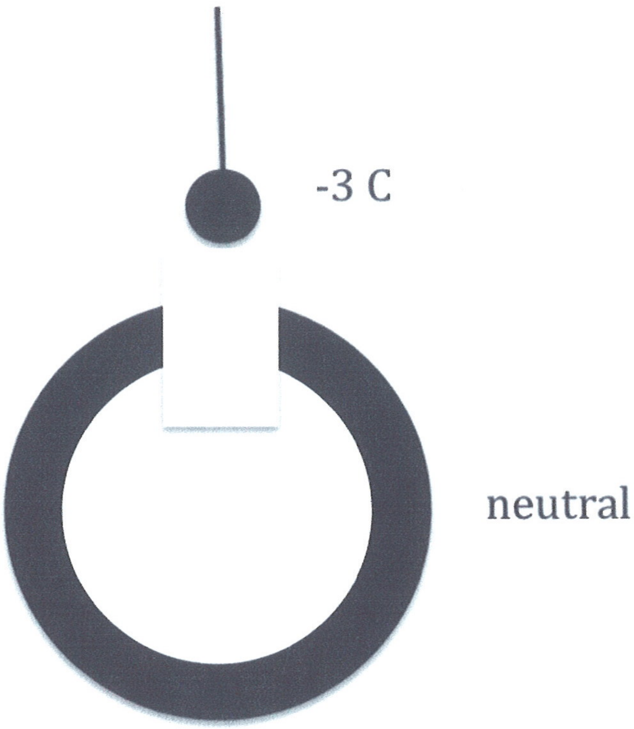
$$V(1,2,2) - 5V = \left[-4y\Big|_1^2 - 8z\Big|_1^2 \right] V$$

$$= \left[-4(2-1) - 8(2-1) \right] V$$

$$= (-4-8)\,V = -12\,V$$

$$V(1,2,2) = -7\,V$$

(10 pts) 10. A small metal sphere is charged to -3 C. It is suspended from a non-conducting string. There is a much larger diameter uncharged conducting sphere with a small hole in the top. The hole is large enough that the smaller sphere can be lowered in without touching the larger sphere.



Part I: The smaller sphere is lowered in without touching the larger sphere.

(2 pts) a) what is the charge on the inside wall of the larger sphere?

3 C

(2 pts) b) what is the charge on the outside wall of the larger sphere?

- 3 C

Part II: The smaller sphere is now lowered till it touches the large sphere and then is raised and removed from the larger sphere without ever touching the larger sphere again.

(2 pts) a) what is the charge on the smaller sphere?

0

(2 pts) b) what is the charge on the inside walls of the larger sphere?

0

(2 pts) c) what is the charge on the outside walls of the larger sphere?

-3 C

(10 pts) 11. Ten charges of value $-\frac{10^{-9}}{36\pi}$ C are arranged on a circle of diameter 1 m centered at the origin. If $V(\infty) = 0$, what is the voltage at V(0,0,0)?

Each charge will make the same contribution to V(0,0,0). So we find V(0,0,0) due to one charge and multiply by 10.

V(0,0,0) due to one charge $= \frac{1}{4\pi\epsilon_0} \frac{\left(-\frac{10^{-9}}{36\pi} C\right)}{1\ m}$

$$= \frac{1}{4\pi\left(\frac{10^{-9}}{36\pi} \frac{F}{m}\right)} \frac{\left(-\frac{10^{-9}}{36\pi} C\right)}{0.5\ m}$$

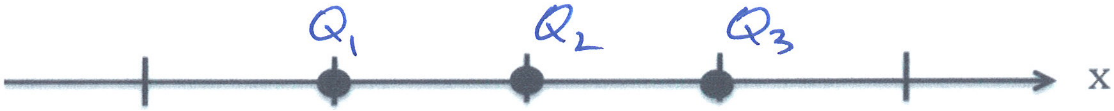
$$= \frac{-1}{2\pi} V$$

so

$$V(0,0,0) = 10\left(-\frac{1}{2\pi} V\right)$$

$$V(0,0,0) = -\frac{5}{\pi} V$$

(10 pts) 12. Three point charges are aligned on the x-axis as shown.



The charges are all $-\sqrt{\dfrac{10^{-9}}{36\pi}}$ C and the separation between charges is 1 m. What is the potential energy of this arrangement of charges?

Bring the charges in from infinity one at a time determining the work involved. The sum of the work to position the charges is the PE

Bring in Q_1 first, this takes no work

The work to now bring in Q_2 is

$$W_2 = Q_2 V_{21} = Q_2 \frac{Q_1}{4\pi\epsilon_0 (1m)} = -\sqrt{\frac{10^{-9}}{36\pi}}\, C \; \frac{-\sqrt{10^{-9}/36\pi}\, C}{4\pi\left(\frac{10^{-9}}{36\pi}\frac{F}{m}\right)(1m)}$$

$$W_2 = \frac{1}{4\pi} J$$

The work to now bring in Q_3 is

$$W_3 = Q_3 V_{31} + Q_3 V_{32} = -\sqrt{\frac{10^{-9}}{36\pi}}\, C \left[\frac{-\sqrt{10^{-9}/36\pi}\, C}{4\pi\left(\frac{10^{-9}}{36\pi}\frac{F}{m}\right)(2m)} + \frac{-\sqrt{10^{-9}/36\pi}\, C}{4\pi\left(\frac{10^{-9}}{36\pi}\frac{F}{m}\right)(1m)}\right]$$

$$W_3 = \frac{1}{8\pi} J + \frac{1}{4\pi} J = \frac{3}{8\pi} J$$

$$PE = W_1 + W_2 + W_3 = 0 + \frac{1}{4\pi} J + \frac{3}{8\pi} J$$

$$= \frac{5}{8\pi} J$$

second method using $W = \frac{1}{2}\sum_{k=1}^{3} Q_k V_k$

so we first need to find V_1, V_2 & V_3

Note $V_1 = V_3$

$$V_1 = \frac{-\sqrt{\frac{10^{-9}}{36\pi}}\,C}{4\pi\epsilon_0(1m)} + \frac{-\sqrt{\frac{10^{-9}}{36\pi}}\,C}{4\pi\epsilon_0(2m)}$$

$$V_1 = \frac{-3\sqrt{\frac{10^{-9}}{36\pi}}\,C}{4\pi\epsilon_0\ m} = V_3$$

$$V_2 = \frac{-\sqrt{\frac{10^{-9}}{36\pi}}\,C}{4\pi\epsilon_0(1m)} + \frac{-\sqrt{\frac{10^{-9}}{36\pi}}\,C}{4\pi\epsilon_0(1m)}$$

$$V_2 = \frac{-\sqrt{\frac{10^{-9}}{36\pi}}\,C}{2\pi\epsilon_0\ m}$$

$$W = \frac{1}{2}\left(Q_1V_1 + Q_2V_2 + Q_3V_3\right)$$

$$W = \frac{1}{2}\left[\frac{3\left(-\sqrt{\frac{10^{-9}}{36\pi}}\,c\right)\left(-\sqrt{\frac{10^{-9}}{36\pi}}\,c\right)}{8\pi\epsilon_0 \ m} + \frac{\left(-\sqrt{\frac{10^{-9}}{36\pi}}\,c\right)\left(-\sqrt{\frac{10^{-9}}{36\pi}}\,c\right)}{2\pi\epsilon_0 \ m} + \frac{3\left(-\sqrt{\frac{10^{-9}}{36\pi}}\,c\right)\left(-\sqrt{\frac{10^{-9}}{36\pi}}\,c\right)}{8\pi\epsilon_0 \ m}\right]$$

$$= \frac{1}{2}\left[\frac{6\left(\frac{10^{-9}}{36\pi}\right)c^2}{8\pi\left(\frac{10^{-9}}{36\pi}\frac{F}{m}\right)m} + \frac{\frac{10^{-9}}{36\pi}c^2}{2\pi\left(\frac{10^{-9}}{36\pi}\frac{F}{m}\right)m}\right]$$

$$= \frac{1}{2}\left[\frac{10}{8\pi}\frac{C^2}{F}\right] = \frac{5}{8\pi}\frac{C^2}{F} = \frac{5}{8\pi} J$$

$$\frac{C^2}{F} = \frac{C^2}{(C/V)} = CV = C\frac{J}{C} = J$$